

SEMESTER-III

COURSE 8: SPECIAL FUNCTIONS

Theory

Credits: 4

5 hrs/week

Learning Outcomes

After successful completion of the course will be able to

1. Understand the Beta and Gamma functions, their properties and relation between these two functions, understand the orthogonal properties of Chebyshev polynomials and recurrence relations.
2. Find power series solutions of ordinary differential equations.
3. Solve Hermite equation and write the Hermite Polynomial of order (degree) n , also Find the generating function for Hermite Polynomials, study the orthogonal properties of Hermite Polynomials and recurrence relations.
4. Solve Legendre equation and write the Legendre equation of first kind, also find the generating function for Legendre Polynomials, understand the orthogonal properties of Legendre Polynomials.
5. Solve Bessel equation and write the Bessel equation of first kind of order n , also find the generating function for Bessel function understand the orthogonal properties of Bessel unction.

Course Content

Unit-1

Beta and Gamma functions, Chebyshev polynomials

Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions.

Another form of Beta Function, Relation between Beta and Gamma Functions.

Chebyshev polynomials, orthogonal properties of Chebyshev polynomials, recurrence relations, generating functions for Chebyshev polynomials.

Unit-2

Power series and Power series solutions of ordinary differential equations

Introduction, summary of useful results, power series, radius of convergence, theorems on Power series

Introduction of power series solution of ordinary differential

equation

Ordinary and singular points, regular and irregular singular points, power series solution.

Unit-3

Hermite polynomials

Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials, generating function for Hermite polynomials. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few Hermite Polynomials. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

Unit-4

Legendre polynomials

Definition, Solution of Legendre's equation, Legendre polynomial of degree n , generating function of Legendre polynomials. Definition of $P_n(x)$ and $Q_n(x)$,

General solution of Legendre's Equation (derivations not required) to show that $P_n(x)$ is the coefficient of h^n in the expansion of $(1 - 2xh + h^2)^{-1/2}$

^{1/2} Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

Unit-5

Bessel's equation

Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n , Bessel's function of the second kind of order n .

Integration of Bessel's equation in series form $x=0$, Definition of $J_n(x)$, recurrence formulae for $J_n(x)$, Generating function for $J_n(x)$, orthogonality of Bessel functions.

Activities

Seminar/ Quiz/ Assignments/ Applications of Special functions to Real life Problem /Problem Solving Sessions.

Text Book

Special Functions by J.N.Sharma and Dr.R.K.Gupta, Krishna Prakashan,

Reference Books

1. Dr.M.D.Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
2. Shanti Narayan and Dr.P.K.Mittal, Integral Calculus, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
3. George F. Simmons, Differential Equations with Applications and Historical Notes, Tata McGRAW-Hill Edition, 1994.
